

AP Physics 1 Investigation 6: Harmonic Motion

What factors affect the motion of a pendulum?

Central Challenge

In this investigation, students explore the motion of a pendulum in two parts. In the first part, students experimentally determine what factors affect the period of a pendulum. In the second part, students create the motion graphs resulting from the periodic motion.

Background

A simple pendulum is a system that can be modeled as a point mass (m) at the end of a string of negligible mass and of length (L). The pendulum executes oscillatory motion because gravity (or more specifically, the component of gravity perpendicular to the string) provides a restoring force that pulls the pendulum back toward equilibrium at every point in its motion. The gravitational force is dependent on the mass of the pendulum bob, and since $\vec{a} = \frac{\Sigma \vec{F}}{m}$, the acceleration of the bob is independent of its mass, and so the period of the pendulum is independent of its mass. For small angles of oscillation, the period is also independent of the amplitude, so the motion approximates simple harmonic motion. So the period of a simple pendulum depends only on its length and the acceleration due to gravity (g).

An example of a system that exhibits simple harmonic motion is an object attached to an ideal spring and set into oscillation. The spring's restoring force depends on the displacement from equilibrium but not on the mass of the object in oscillation. The period can be shown to be equal to $T = 2\pi\sqrt{m/k}$ if the mass of the spring can be neglected. Refer to any calculus-based introductory physics textbook for the derivation of this period (and for a pendulum) from a second-order linear differential equation.

Real-World Application

The most obvious real-world application of harmonic motion for students is the idea of time keeping. Everything from traditional grandfather clocks to atomic clocks use periodic oscillations to keep time. For those who study music, metronomes are a type of pendulum that keeps time. A child on a swing in the playground is a reasonable approximation of a simple pendulum, assuming he or she does not swing too high (i.e., at too great an amplitude). A good discussion could be had about under what conditions a child on swing acts like a simple pendulum, and under what conditions he or she does not. The period of an old-fashioned metronome depends on the position of the mass on the vertical post. Having students first visualize these types of oscillations is a useful way to start discussion about this investigation.

Inquiry Overview

In this investigation, students explore the motion of a pendulum in two parts.

In Part I, students experimentally determine which quantity or quantities affect the period of a pendulum. This part of the lab can be more open inquiry if implemented at the start of a simple harmonic motion unit; or it can have more structured, guided inquiry if implemented as the first lab (or very early in the course), in accordance with the modeling curriculum out of Arizona State University (see Supplemental Resources). As the first lab it would then serve to teach students about designing experiments, making measurements, and calculating or estimating uncertainties.

In Part II, students create the harmonic motion graphs resulting from the periodic motion. This part of the lab can either be open inquiry, allowing students to determine how to graph the motion of the pendulum as a function of time, or it can be more guided inquiry depending on the experience and sophistication of your students at the time you decide to implement this lab.

Connections to the AP Physics 1 Curriculum Framework

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding

3B Classically, the acceleration of an object interacting with other objects can be predicted by using $\vec{a} = \frac{\Sigma \vec{F}}{m}$

Learning Objectives

3.B.3.1 The student is able to predict which properties determine the motion of a simple harmonic oscillator and what the dependence of the motion is on those properties. (Science Practice 6.4)

3.B.3.2 The student is able to design a plan and collect data in order to ascertain the characteristics of the motion of a system undergoing oscillatory motion caused by a restoring force. (Science Practice 4.2)

3.B.3.3 The student can analyze data to identify qualitative or quantitative relationships between given values and variables (i.e., force, displacement, acceleration, velocity, period of motion, frequency, spring constant, string length, mass) associated with objects in oscillatory motion to use that data to determine the value of an unknown. (Science Practices 2.2 and 5.1)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

Skills and Practices Taught/ Emphasized in This Investigation

Science Practices	Activities
2.2 The student can <i>apply mathematical routines</i> to quantities that describe natural phenomena.	Students graph the period as a function of mass, angle, and length. Students derive an equation relating the period of a pendulum to the length of the pendulum using their data.
4.2 The student can <i>design a plan</i> for collecting data to answer a particular scientific question.	Students design a plan for collecting data to determine what factors affect the period of a simple pendulum.
4.3 The student can <i>collect data</i> to answer a particular scientific question.	Students collect data while varying several factors (mass, length, angle) to determine which affect the period of a pendulum and how they affect it.
5.1 The student can <i>analyze data</i> to identify patterns or relationships	Students analyze the data for period vs. mass, length, and angle to identify which factors affect the period of a pendulum and to determine the mathematical relationship from the data.
6.4 The student can <i>make claims and predictions about natural phenomena</i> based on scientific theories and models.	Students predict the period of a pendulum based on its length, mass, and angle of release.

[NOTE: Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group:

Part I:

- ▶ String
- ▶ Set of calibrated masses (20–500 g)
- ▶ Stopwatch or timer
- ▶ Meterstick
- ▶ Protractor
- ▶ Support rod
- ▶ (Optional) Pendulum clamp

Part II:

- ▶ Paper and tape (to create a scroll)
- ▶ Leaking bob (can be made by placing a paper towel in a small funnel and soaking it with colored water)
- ▶ (Optional) Constant speed buggy
- ▶ (Optional) Motion detector, software, and computer
- ▶ (Optional) Video camera and analysis software

Extension:

- ▶ Spring

Timing and Length of Investigations

- ▶ **Teacher Preparation/Set up:** 10–15 minutes
- ▶ **Student Investigation:** 170 minutes
 - ▶ **Part I:** 85 minutes
 - Prelab/demonstration/discussion: 10 minutes
 - Student-centered investigation: 45 minutes
 - Student-led discussion of results: 15 minutes
 - Postlab data linearization activity and discussion: 15 minutes
 - ▶ **Part II:** 85 minutes
 - Prelab/demonstration/discussion: 10 minutes
 - Student-centered investigation: 45 minutes
 - Student-led discussion of results: 15 minutes
- ▶ **Postlab Discussion:** 15 minutes
- ▶ **Total Time:** approximately 3.5 hours

Safety

General lab safety should be observed. Instruct students to exercise special caution as they swing masses in the classroom. Make sure all members of the group are clear of the swinging area and are aware when the pendulum will be released.

In addition to student safety, motion sensors and other equipment should be protected when using springs. If you decide to do the extension to this lab, warn students to keep attached masses small enough and amplitudes small enough that springs are not extended beyond their elastic limits. Also, if motion sensors are used, they should be protected from swinging pendulums or masses falling from springs.

Preparation and Prelab

Part I:

This part of the investigation works well for introducing the skills of taking data and modeling the behavior of an object graphically and mathematically.

If used at the beginning of the course, it will probably be necessary to discuss how to reduce uncertainties in timing measurements. You can start with a short pendulum, around 30 centimeters long, and distribute the stopwatches among students in the class. Then, as a class, time one complete oscillation of the pendulum. Record the values and have students calculate the average and uncertainty in the period. Given that the period is less than 1 second, human reaction time error will be a large percentage of the period. Have the class time ten complete oscillations. If all goes well, the uncertainty in the time measurement will be the same, but the percentage uncertainty of the total time will decrease. For example, 0.25 second reaction time uncertainty is 25 percent of a 1.0 second period measurement, but only 2.5 percent of a 10 second measurement of ten oscillations of the same pendulum.

How much instruction you give students before the lab will depend on when you choose to implement it. If this lab is done as a first lab, it might be useful to have a class discussion brainstorming factors that might affect the period of a pendulum. Since this is an easy system to analyze, students have many ideas about what might affect the period. They will make suggestions like “the force with which you push it to start the motion” and “air resistance.” This can lead to a discussion about how to make measurements, what we can actually measure, and what factors we can control. Air resistance may play a role, but at the speeds of these pendula, it is not large, nor can it be controlled or eliminated. A discussion of measuring the initial force should lead students to realize that this is not easy to measure, and it is directly related to the amplitude of the pendulum, which is much more easily measured and thus controlled.

A class discussion such as this can help new physics students narrow the field of possibilities to mass, length, and angle as factors that may affect the period of a pendulum. This narrows the scope for them and makes the task more manageable. If you choose to implement this lab later in the year when students will have more experience designing labs, then you may wish to skip the class discussion and let students decide for themselves how to narrow the scope of the investigation. Depending on how much time you have for this lab, you may also choose to have several groups study the effect of angle on period, several groups study mass, and several groups study length; then have the groups share the data and results with each other.

Part II:

If Part I is done as the first experiment of the year, then this part can be delayed until the oscillations unit, which should come after the kinematics and forces units. Part II works well at the beginning of a unit on simple harmonic motion. At that time, you can revisit the period vs. length experiment and have students do a more mathematical analysis.

The Investigation

Part I:

Start with a demonstration of a pendulum about 30–50 centimeters long. Pull the pendulum bob back and release, and catch the bob when it returns. Explain to students that the time for the pendulum to complete one complete cycle is referred to as the *period*. Next, pose the question, “What factors affect the period of a pendulum, and what factors do not?” You could also phrase the question as, “What could we change about this system that would change the time it takes the pendulum to swing back and forth once?”

At this point, you could choose to have a class discussion or release them to their groups to discuss and plan their data-taking strategies. If you choose to have a whole-class discussion, help students focus on what can be measured and what tools they will use to measure the quantities they decide to measure. This would be a good time to discuss the benefits of timing multiple periods. Students should also refrain from having multiple students involved in the timing such that one person says “start” and another person releases the pendulum. To reduce uncertainty, the pendulum should be set in motion, and then the student with the stopwatch starts timing at some point in the motion, and stops when it returns to that position after multiple periods. It is up to you how much guidance you want to give before releasing students to their groups to design and execute their plan.

As you circulate, remind students to manipulate one variable at a time and record their data neatly in tables, and encourage students to display the data in the best way to represent the relationship. Students will usually choose to vary the mass of the pendulum, the angle of release from the equilibrium position, and the length of the pendulum. Consideration needs to be taken as to where to measure the length of the pendulum: the top of the bob, the middle, or the bottom. You might want to provide some guidance by reminding them a simple pendulum models the object as a point mass and asking them where the point would be (center of mass).

Most guidance in an inquiry lab should take the form of questions to students as to what they are doing and why they are doing it that way. Make them articulate what they know about best scientific practice, and remind them to engage in that.

At the conclusion of the first part of this investigation, students should observe, from their data, a significant relationship between length and time (period). They may assume that the mass affects the period as well, given that they are not likely to get the exact same value for the period for each different mass. At this point, encourage students to plot the period as a function of mass and period as a function of length and observe the results. As students analyze the relationships, they should ultimately linearize the data in order to determine the relationship between time (period) and length.

Have students present their results to the class (or otherwise share and discuss them) before proceeding to the next part.

Part II:

To start this part of the investigation, ask students to consider, “How could we graphically express periodic motion?”

Challenge students to draw a prediction of the position-vs.-time graph of the motion of the pendulum for one full period. Suggest the equilibrium position to be where $x = 0$. Once students have completed their graph prediction, instruct students to design an experiment that allows them to directly record position and time.

This part may be very challenging to many students. Allow each group enough time to discuss and brainstorm some ideas for collecting this data. Students may choose to use a motion detector to collect this data or, depending on their level of sophistication, video analysis. A lower-budget alternative is a leaking bob, which drops colored liquid onto a moving piece of paper, although this introduces a small error in the length of the pendulum and will concern those students who believe mass is a factor.

Students should design a method of tracing the position of the swinging pendulum bob (the leaking bob) onto a constantly moving scroll/paper. Remind them to use small amplitude, based on their results from Part I. Encourage them to think carefully about how to move the paper at a constant rate.

Extension

Another example of periodic motion is an object oscillating at the end of a spring. Ask students to determine mathematically the relationship between period and mass for an object oscillating at the end of a spring hung vertically from a support rod. Depending on when you choose to implement this lab, you could hand students a spring and a stopwatch and ask them to find the spring constant of the spring. In this case, they should already know the relationship, $T = 2\pi\sqrt{m/k}$ where m is the mass of the object and k is the spring constant. They can compare the spring constant obtained using the slope of a graph of T^2 vs. mass to one using the relationship using $F = k\Delta x$, where F is the force applied to the spring and Δx is the spring extension from equilibrium.

Common Student Challenges

Part I:

One common challenge for students is how to measure the length of the pendulum, and how to keep the length of the pendulum constant while varying mass, angle, or other factors they might choose to study. If a set of hooked masses is used, the different masses will have different heights, and thus the length of the pendulum from support point to the bottom of the mass will vary by up to 5 centimeters depending on which masses are used. This will most likely present itself in a slight increase in the period of a pendulum with increasing mass. Address this uncertainty with each group individually as you circulate, or address it in a class discussion when students present their results.

A robust discussion of measurement uncertainty can now take place. Some guiding questions for this discussion include:

- ▶ How much longer was the pendulum with the 500-gram mass compared to the 50-gram mass?
- ▶ Was it 10 times longer or only about 5 percent longer?
- ▶ Was the length of the pendulum really constant when the mass was varied?
- ▶ To which part of the hooked mass should you measure when you measure the length of a pendulum?
- ▶ If the length of the pendulum was 30 cm, and the 500-gram mass was 5 cm taller, how much uncertainty does this introduce?

Another common student challenge relates to timing. Students regularly time by having one student watch the motion and say “start” and “stop,” while another student operates the stopwatch. Students should learn that the person operating the stopwatch is the one who observes the motion and counts the oscillations. This results in less human reaction-time error. Sometimes timing demonstrations work to show students the added uncertainty. It might suffice to tell them that each time one student says “start” and the other one reacts, they introduce more reaction-time uncertainty. A good point of discussion may be whether to measure from the bottom of the arc or the top of the arc and why.

When graphing, a common mistake students make is to limit the vertical axis range on graphs to the range of data collected. If they do this, in particular for the period-vs.-mass data, they will miss the fact that the period is independent of the mass. During the analysis portion, encourage students to start each axis at zero and continue beyond the greatest value of their data.

Part II:

If students have a difficult time drawing a position vs. time graph for the pendulum, ask them if they have ever seen a polygraph (lie detector) or seismograph record data (see Supplemental Resources for videos of these devices). Both of these have a piece of paper moving under a needle that writes. Have students imagine a pen attached to the pendulum that writes on a moving piece of paper.

Marking the position of the pendulum on the paper becomes the second challenge. Some will want to attach a marker to the pendulum to draw on the paper; however, this causes both friction and incomplete data, because the marker will not keep contact with the paper as it swings. The “leaking bob” will mark the position without adding external forces to the system. If students find it challenging to pull the paper scroll at a constant rate, suggest attaching it to a constant speed buggy (available online for under \$10).

Analyzing Results

Part I:

Due to the investigative nature of this experiment, having students report on individual large whiteboards is ideal (or large poster/bulletin board paper). The whiteboards are useful for displaying procedure, data, and graphs in a way that they may be easily shared. Having students sketch a graph of their data in a large format that can be shared during the discussion will give students the opportunity to see how others approached the investigation.

The following are prompts for discussion as students analyze their results. It is up to you which factor to address first. If the students agree that length affects period, then they might see that the objects with more mass are longer/taller and thus add to the length of the pendulum. Thus, small variations in period as the mass is changed could then be attributed to the variations in sizes of the different objects used to vary the mass.

1. *Does mass matter?* Many students will expect that it should. Students will probably obtain slightly different periods for different masses, and some will think these differences are “real.” This is an opportunity, not a problem. After the small groups have presented their results, challenge the whole class — including groups that measured other quantities — to take the shared data and make the best argument they can that period depends on mass, or that it doesn’t depend on mass. This could lead to an authentic discussion about uncertainty. In particular, as mentioned above, students should consider whether, when changing mass, the length of the pendulum changed significantly.

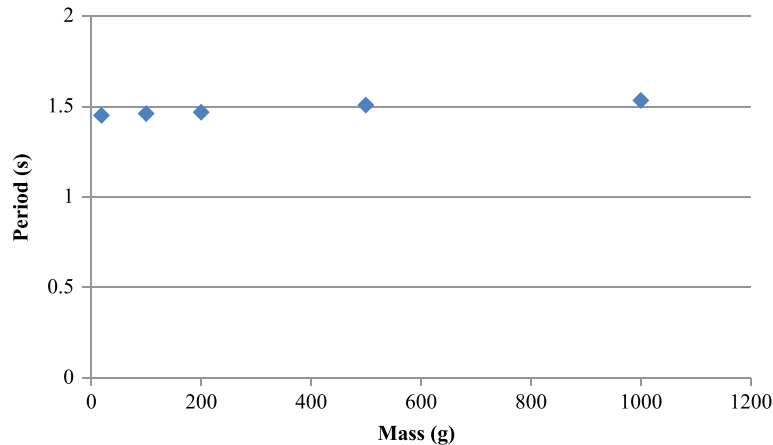
Sample student data for this part of the experiment is as follows:

Mass (g)	Period (s)
20	1.451
100	1.46
200	1.473
500	1.515
1000	1.542

Table 1

Students frequently claim that the period for the 1000-gram mass is greater than that for the 20 grams, and thus the mass affects the period. This is when to ask the question, “You increased the mass by a factor of 50 (for example), by how much did this increase the period?”

A graph of the data yields the following:



Graph 1

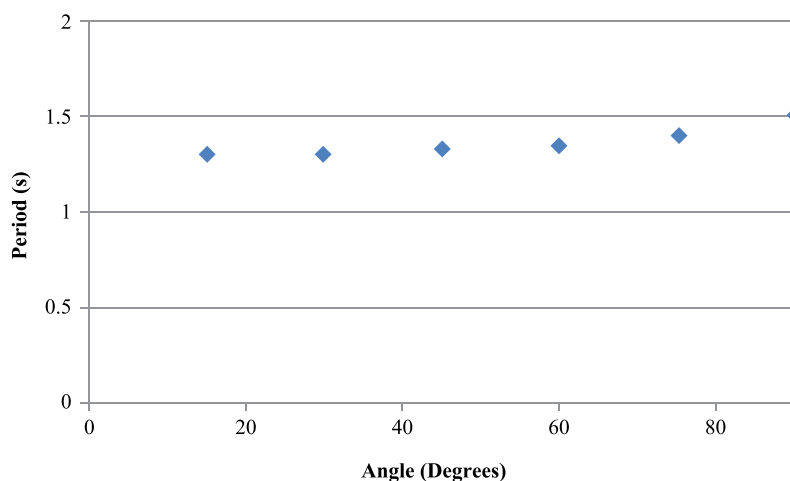
Groups using graphical representations will make the most convincing arguments, and you can turn this into a meta-discussion: What way of showing the data best clarifies whether the differences between the measured periods for different masses are significant vs. the result of measurement uncertainty? One source of measurement uncertainty that comes in to play if students are using hooked masses is that that larger hooked masses are much taller than the smaller hooked masses. Thus, if the students merely keep the string length constant while changing masses, they will be inadvertently changing the length as well as the mass when they change to larger masses. This can be pointed out to explain the slight increase in period for larger masses. Or students could be advised to keep the length the same and always measure to the center of mass of the pendulum bob.

2. *Does angle matter?* This discussion can go the same way as the discussion above, except now ask students from all small groups to represent the data in a way that best helps decide the issue, as decided in question #1 above. This discussion could also take advantage of another teachable moment regarding conceptual learning. Students will likely come to consensus that the angle doesn't matter, or only matters a little for larger angles, but they will likely find this result counterintuitive.

Typical data for this section:

Angle (degrees)	Period (s)
15	1.308
30	1.305
45	1.335
60	1.35
75	1.404
90	1.512

Table 2



Graph 2

Students can see from this graph that for angles less than about 30 degrees, the period is relatively constant; but as the angle increases, the period increases. The mathematics of the dependence of period on angle beyond 30 degrees is too complex for students at this level, so instruct them to make sure that for future pendulum measurements, as long as the angle is less than 20–30 degrees, the period is relatively constant. If you wish for more precision in your students' data, instruct them to do further investigations of the dependence of period on angle. Specifically, they could measure the period for many different angles between 0 and 30 degrees to see the variation in that range. The sine of an angle (in radians) is within 10 percent of the angle for angles less than 30 degrees, and within about 2 percent for angles less than 20 degrees.

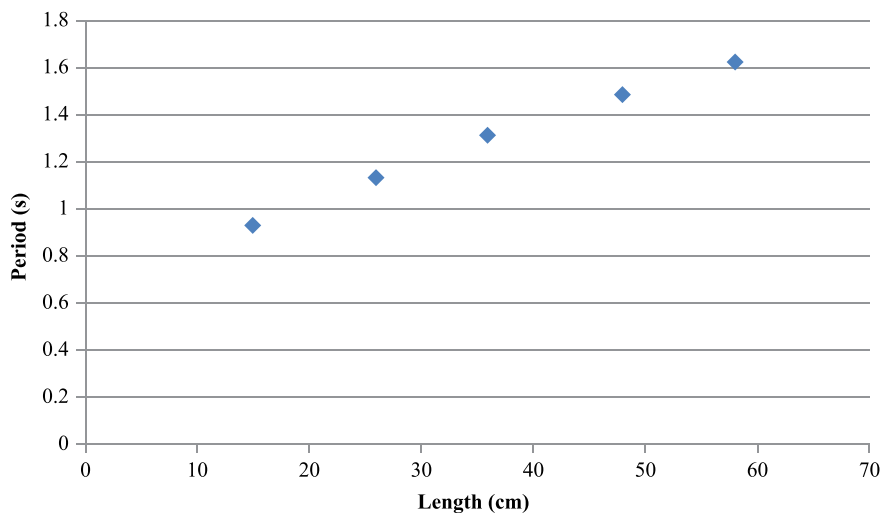
It is a useful exercise to have students put their calculator in radian mode and compare the sine of an angle to value of that angle for angles less than one radian. For example, 20 degrees is equal to 0.35 radians. The sine of 0.35 radians is 0.343, which is approximately 2 percent smaller than 0.35.

3. *Does length matter?* Since students will quickly agree that string length *does* matter, the discussion can quickly transition to focus on figuring out what the relation is. Is it linear, square, square root, or something else? At this point a discussion of straightening graphs is imperative, if it has not already been done. Once again, the “Modeling Instruction” website has excellent resources for this discussion.

Sample data for this section:

Length (cm)	Period (s)
15	0.93
26	1.14
36	1.32
48	1.49
58	1.63

Table 3



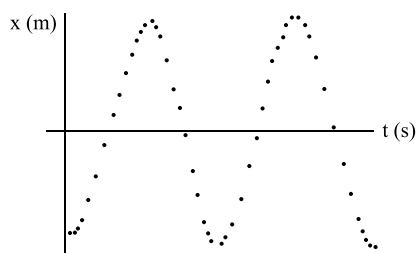
Graph 3

The range of the data shown above is very small, and demonstrates the fact that, for small ranges of data, the difference between a straight line and a square-root curve can be difficult to see. Encourage students to take a much larger range of data if their results look like Graph 3. As an alternative, lead a class discussion and time the period of a pendulum that is much longer (1.5–2 meters or more depending on the height of your classroom). This data can then be added to the data set to more clearly illustrate the nature of the curve. It is difficult to time the period of a pendulum shorter than 10 centimeters, since even the smallest masses are 2–3 centimeters tall. However, it might be worth attempting, in order to extend the data range even more; and the length measurements can be made more accurate by measuring to the center of mass of the pendulum bob in each case.

Students should then linearize the data by plotting one of the following: period vs. square root of length or period squared vs. length. From this graph, the mathematical relationship between period and length can be determined. Depending on when you decide to implement this lab, you can continue with a comparison to the equation for the period of a simple pendulum, $T = 2\pi\sqrt{L/g}$. You might want to wait to do this until the unit on simple harmonic motion. When this is done, students use the slope of their linearized graph to calculate the numerical value of g and compare to the accepted value. Additional discussions about uncertainty and whether they got the “right” answer can occur at this point as well.

As a summative assessment, ask students to use their data to make predictions about the period of a pendulum with a given mass, angle, and length. They can either interpolate/extrapolate from their graphical data, or use the equation they obtained for period vs. length to calculate a value. Make sure the angle you provide for them is less than 30 degrees.

Part II:



Graph 4

How you proceed with the analysis of this section depends on when you implement this lab. If Part I of this lab is implemented at the beginning of the course, it is necessary to wait until the study of oscillatory motion, after the study of forces and kinematics, to complete Part II.

Have students consider the following:

- ▶ Does their position vs. time graph match the predictions they made, and if not, why not?
- ▶ What is different about the motion, and what is the same as their prediction?
- ▶ What role does uncertainty play in their graphs?

Once an acceptable graph of the position vs. time of the pendulum has been established, as shown in Graph 4, students should use this graph to sketch a graph of velocity vs. time. If they struggle with this, remind them of their knowledge of kinematics and the relationship between position vs. time graphs and velocity vs. time graphs.

Some of the tasks you can ask students to do include:

- ▶ Explain the relationship between the velocity and the slope of the position vs. time graph.

- ▶ Identify when and where the bob reaches the maximum and minimum velocities.
- ▶ Explain how to construct the acceleration vs. time graph from the velocity vs. time graph and locate when and where maximum and minimum accelerations occur.

Once they have made these predictions, they check their predictions using a motion detector and computer interface (assuming this equipment is available). Ask them to label all the points of zero speed on both the position graphs and velocity graphs and to comment on similarities between the multiple points. They should comment on the relationship between the acceleration graph and the position graph. Students should notice that the acceleration graph is the negative of the position graph.

Assessing Student Understanding

Part I:

After completing this investigation, students should be able to:

- ▶ Design an experiment to determine the effect of mass, angle, and length on the period of a pendulum;
- ▶ Measure the period of a pendulum by timing multiple oscillations; and
- ▶ Determine the relationship between mass, angle, or length and the period of a pendulum by examining data in the form of tables or graphs.

Part II:

Students should also be able to:

- ▶ Graph the position of a pendulum as a function of time;
- ▶ Determine an equation relating the period of a pendulum to its length;
- ▶ Predict the period of a simple pendulum given the length, mass, and release angle; and
- ▶ Draw the graph of velocity vs. time and acceleration vs. time from their graph of position vs. time.

Assessing the Science Practices

Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

Proficient	Uses and applies mathematical routines to detect and describe patterns in the data, and compares the period of a pendulum in terms of its length, mass, and/or amplitude (in terms of angle).
Nearly Proficient	Uses and applies mathematical routines that describe the patterns in the period of a pendulum in terms of its length with only occasional or minor errors.
On the Path to Proficiency	Uses and applies mathematical routines to describe the period of a pendulum in terms of its length with some inconsistency and/or errors.
An Attempt	Incorrectly identifies patterns in the mathematical data or incorrectly applies routines to describe them, and description contains major errors.

Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Designs a plan that will allow a determination of the factors that affect the period of a pendulum.
Nearly Proficient	Designs a plan for measuring the period of a pendulum in terms of angle but cannot articulate how that plan will lead to a rule for the period.
On the Path to Proficiency	Designs a plan to measure the period of a pendulum but it's not clearly defined or articulated — it doesn't take into account varying length, mass, or angle.
An Attempt	Presents an incomplete design for a plan that attempts to measure period as a function of other variables; makes errors in identifying variables.

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects appropriate, adequate, and accurate data in a methodical way, and presents the data in an organized fashion.
Nearly Proficient	Collects appropriate and adequate data; some minor errors are present, and/or the presentation is logical but lacking in an organized format.
On the Path to Proficiency	Collects inadequate or irrelevant data with significant gaps or errors, and presents data in a way that is disorganized and lacks logic.
An Attempt	Collects irrelevant, inaccurate, or incomplete data and doesn't provide any organization for this data.

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Constructs a graph to analyze the data and accurately determine the effect of mass, angle, and length on the period of a pendulum. Uses the graph of period squared vs. length to derive a mathematical relationship between period and length.
Nearly Proficient	Constructs a graph to analyze the data and qualitatively determine the effect of mass, angle, and length on the period of a pendulum, but unable to derive a mathematical relationship between period and length.
On the Path to Proficiency	Identifies patterns in the data for the period of a pendulum, but unable to form a complete conclusion from this analysis.
An Attempt	Forms some accurate analysis of the graphs of the period of a pendulum, but unable to come to an accurate conclusion.

Science Practice 6.4 The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

Proficient	Predicts the period of a pendulum accurately from a graph of period squared vs. length or period vs. square root of length.
Nearly Proficient	Uses a graph to make a prediction, but fails to take the square root of the period.
On the Path to Proficiency	Makes estimates of the period of a pendulum based on data, but cannot make accurate calculations using an equation or a graph.
An Attempt	Makes incorrect predictions about the period of a pendulum using the data collected and graphed.

Supplemental Resources

Carvalhoes, Claudio G., and Patrick Suppes. "Approximations for the Period of the Simple Pendulum Based on the Arithmetic-Geometric Mean." *American Journal of Physics* 76, no. 12 (2008): 1150–1154. [*This article discusses methods for approximating the period for large angles; a resource for the teacher only, and only if the teacher enjoys a good challenge.*]

"Cut the Rope Trailer." YouTube. Video, 1:22. Accessed September 1, 2014. <http://www.youtube.com/watch?v=8xPUdFaraoQ&>. [*This trailer for a "Cut the Rope" video has several good instances of pendulum motion.*]

"How a Seismograph Works." YouTube. Video, 1:04. Accessed September 1, 2014. <http://www.youtube.com/watch?v=Gbd1FcuLJLQ>. [*Good video of how a seismograph works.*]

“I Didn't Know That - Beating a Lie Detector Test.” National Geographic. YouTube. Video, 4:37. Accessed September 1, 2014. <http://www.youtube.com/watch?v=JcDr7O-Wmuk>. [*A video describing the pendulum motion and marking of a Lie Detector Machine.*]

Kuhn, Jochen, and Patrik Vogt. “Analyzing Spring Pendulum Phenomena with a Smart-Phone Acceleration Sensor.” *The Physics Teacher* 50, no. 8 (2012): 504. [*Alternative methods for measuring the properties of a pendulum.*]

“Modeling Instruction.” Arizona State University. Accessed September 1, 2014. <http://modeling.asu.edu/>.

Mires, Raymond W., and Randall D. Peters. “Motion of a Leaky Pendulum.” *American Journal of Physics* 62, no. 2 (1997): 137–139.

“Properties of Periodic Motion.” The Physics Classroom. Accessed September 1, 2014. <http://www.physicsclassroom.com/class/waves/u10l0b.cfm>. [*This website is a good source for properties of periodic motion.*]

“Simple Pendulum.” Walter Fendt. Accessed September 1, 2014. <http://www.walter-fendt.de/ph14e/pendulum.htm>.