

# AP Physics 1 Investigation 3: Circular Motion

How do you determine the period of a conical pendulum?

## Central Challenge

In this investigation, students use a toy that executes motion in a conical pendulum to study circular motion. Given only a meterstick and a stopwatch, they must design a procedure and make measurements to predict the period of motion of the conical pendulum.

## Background

A conical pendulum consists of an object moving in uniform circular motion at the end of a string of negligible mass (see Figure 1). A free-body diagram of the object is shown in Figure 2.  $F_T$  represents the tension in the string and the gravitational force on the object is  $mg$  where  $m$  is the object's mass and  $g$  is the acceleration due to gravity.

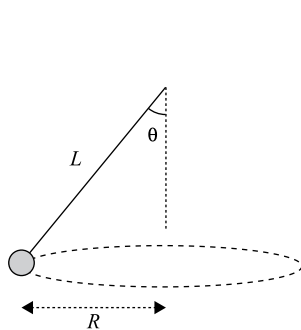


Figure 1

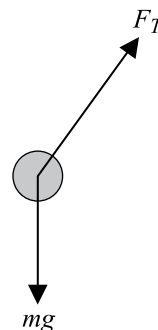


Figure 2

The circular motion of the object is in the horizontal plane, so the horizontal component of the tension is serving as the centripetal force. Since there is no vertical motion of the object, the vertical component of the tension is equal to the gravitational force on the object. In equation form:

$$F_T \sin \theta = ma_c$$

$$F_T \sin \theta = m \frac{v^2}{R}$$

and

$$F_T \cos \theta = mg$$

where  $R$  is the radius of the object's motion,  $v$  is the speed, and  $\theta$  is the angle the string makes with the vertical, as shown in Figure 1.

Combining these equations we get:

$$\tan \theta = \frac{v^2}{gR}$$

The speed of an object in circular motion is given by  $v = \frac{2\pi R}{T}$  where  $T$  is the period of the circular motion. Substituting this relationship into the equation above and rearranging we get  $T^2 = \frac{4\pi^2 R}{g \tan \theta}$ .

Thus, by measuring only lengths such as  $L$  and  $R$  (see Figure 1), and using them to calculate the angle from the vertical, students can predict the period of a conical pendulum.

[NOTE:  $L$  is the length of the pendulum, as measured from the point of attachment of the string to the center of mass of the object at the end of the pendulum (assuming the string has negligible mass), and  $R$  is measured from the center of the circle to the center of mass of the object.]

## Real-World Application

There are many real-world applications of circular motion dealing with interchanges, intersections, and driving a car in general. You can talk about various amusement park rides as well — roller coasters deal heavily with circular motion. The swing ride is an example of a conical pendulum in which the riders sit in swings and move in circular motion around a central support structure (see Figure 3). Other rides, such as the rotor ride, Enterprise wheel, and Ferris wheel, spin the rider in circular motion either horizontally or vertically. NASA uses circular motion in a centrifuge to simulate the high  $g$ -forces on astronauts in flight. Medical equipment such as the centrifuge use circular motion principles to separate out components in test tubes.



Figure 3

## Inquiry Overview

This investigation is a guided inquiry in which students make measurements with a meterstick and use them to predict the period of a self-propelled mass, such as a flying airplane (or flying pig or cow), that moves like a conical pendulum. This is a new twist on what is a familiar lab (see “Circular Motion Studies with a Toy Airplane” in Supplemental Resources).

As part of their experimental design, students should also plan to make multiple measurements to determine or verify the relationship between the length of the pendulum and the angle the string makes with the vertical as the object executes circular motion. They can vary the length and plot graphs of period vs. length, speed vs. length, and angle vs. length, and compare the graphical results to the theoretical results derived using Newton’s second law.

## Connections to the AP Physics 1 Curriculum Framework

**Big Idea 3** The interactions of an object with other objects can be described by forces.

Enduring Understanding	Learning Objectives
<p><b>3.B</b> Classically, the acceleration of an object interacting with other objects can be predicted by using <math>\vec{a} = \frac{\sum \vec{F}}{m}</math>.</p>	<p><b>3.B.1.1</b> The student is able to predict the motion of an object subject to forces exerted by several objects using an application of Newton’s second law in a variety of physical situations with acceleration in one dimension. (Science Practice 6.4)</p> <p><b>3.B.1.2</b> The student is able to design a plan to collect and analyze data for motion (static, constant, or accelerating) from force measurements and carry out an analysis to determine the relationship between the net force and the vector sum of the individual forces. (Science Practices 4.2 and 5.1)</p> <p><b>3.B.2.1</b> The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. (Science Practices 1.1, 1.4, and 2.2)</p>
<p><b>3.E</b> A force exerted on an object can change the kinetic energy of the object.</p>	<p><b>3.E.1.3</b> The student is able to use force and velocity vectors to determine qualitatively or quantitatively the net force exerted on an object and qualitatively whether kinetic energy of that object would increase, decrease, or remain unchanged. (Science Practices 1.4 and 2.2)</p>

**Big Idea 4** Interactions between systems can result in changes in those systems.

### Enduring Understanding

**4.A** The acceleration of the center of mass of a system is related to the net force exerted on the system, where  $\bar{a} = \frac{\Sigma \vec{F}}{m}$ .

### Learning Objectives

**4.A.2.1:** The student is able to make predictions about the motion of a system based on the fact that acceleration is equal to the change in velocity per unit time, and velocity is equal to the change in position per unit time. (Science Practice 6.4)

**4.A.3.1:** The student is able to apply Newton's second law to systems to calculate the change in the center-of-mass velocity when an external force is exerted on the system. (Science Practices 2.2 and 5.1)

[**NOTE:** In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

## Skills and Practices Taught/Emphasized in This Investigation

### Science Practices

**1.1** The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

**1.4** The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

**2.2** The student can *apply mathematical routines* to quantities that describe natural phenomena.

**4.2** The student can *design a plan* for collecting data to answer a particular scientific question.

**4.3** The student can *collect data* to answer a particular scientific question.

**5.1** The student can *analyze data* to identify patterns or relationships

**6.4** The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

### Activities

Students draw free-body diagrams of the object as it executes circular motion.

Students use the free-body diagram and Newton's second law to write equations related to the motion of the object.

Students use equations derived from Newton's second law to analyze the motion of the object.

Students design a plan to use only length measurements to predict the period of a conical pendulum.

Students make measurements of various lengths associated with the motion of the object as it moves in a circle.

Students apply mathematical routines to choose data that will allow them to predict the period of the object's motion. Students analyze the uncertainty in their measurements and make adjustments to reduce these uncertainties where possible.

Students use Newton's second law and length measurements to predict the period of an object moving in a circle.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

## Equipment and Materials

*Per lab group (two to four students):*

- ▶ Battery-operated toy airplane (or flying pig or cow — see Figure 4) with new 1.5-volt AA cells installed
- ▶ Meterstick
- ▶ Stopwatch (for verification only)
- ▶ (Optional) Extra sets of AA cells for the plane that have been drained so they are not at full operating potential difference. [**NOTE:** The cells in the sets should be less than 1.5 V each under load, but each cell in the set of two should be at the same potential.]
- ▶ (Optional) Multimeter to test electric potential difference of each cell

[**NOTE:** Ceiling-suspended, battery-operated airplanes (9-inch wingspan, two AA batteries required) can be obtained from The Physics Toolbox — see Supplemental Resources.]

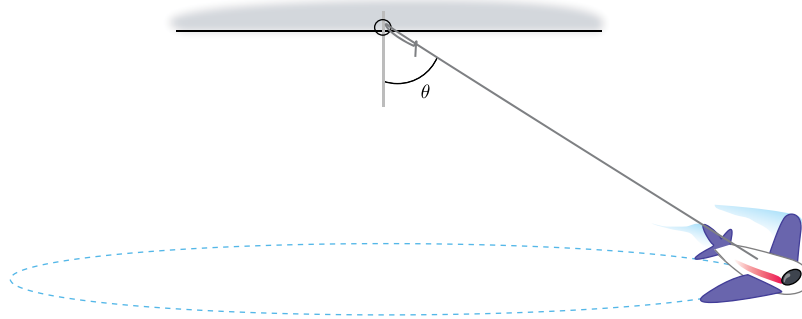


Figure 4

## Timing and Length of Investigation

- ▶ **Teacher Preparation/Set-up:** 15 minutes

The toys need to be suspended so they can execute circular motion — extend them from the ceiling or from a tall stick or pole. You should do this setup prior to the lab. [**NOTE:** Strong hooked magnets can be attached to ceiling metal cross grids to support the swivel hook that comes with the flying toy. Avoid attaching the devices to the ceiling on or at the corners of light fixtures or on sprinkler system apparatus.]

- ▶ **Prelab:** 10 minutes

To demonstrate the conical pendulum, put students in groups and pose the problems to them.

▶ **Student Investigation:** 45–60 minutes

Students design a plan to make measurements, and make the measurements and calculate the period.

▶ **Postlab Discussion:** 15–30 minutes

Students present their results, and share the method they used to predict the period.

▶ **Total Time:** 1.5–2 hours

## Safety

All general safety guidelines should be observed. In addition, some toy airplanes have small plastic propellers that rotate rapidly; students must take care to keep their fingers away from the propellers. Students should also not walk around too much to avoid getting hit in the head by a conical pendulum. Students should be wearing safety goggles on the off-chance that a string breaks.

To prevent students from climbing up on tables or chairs to change ceiling connections, it may be wise to preinstall multiple devices with new cells and with different lengths; then students can take multiple trials by simply moving to a different pendulum (assuming they all are constructed similarly).

## Preparation and Prelab

This lab is best implemented at the end of the circular motion unit and used as a review. Students should already have solved many problems involving circular motion. They should be able to draw a free-body diagram and identify the radius of an object's motion.

Demonstrate for students how to start the toy airplane flying in circular motion. All that remains after that is to present them with this task: using only a meterstick, make measurements that allow for calculation of the period of the plane's motion. Students should not be shown the derivation above in the Background section; rather, they should be required to complete it themselves and decide what measurements to make.

## The Investigation

Students should work in groups of two to four. The number of students per group depends upon how many toy airplanes are available or the time available for groups to rotate through using the setup. Each group should have direct access to a device.

Each group designs and executes a plan for taking measurements with a meterstick to calculate the period of a conical pendulum. They then measure the period with a stopwatch and compare the stopwatch measurement to their prediction.

Some groups will start to measure before they have a plan. Some groups will ask if they can find the mass of the plane. They should not be allowed to use a balance to find the mass of the plane. If they can find the mass of the plane with only a meterstick (no other masses, etc.) then that's fine, but the only measurement tool they are allowed is a meterstick.

Circulate among the groups and encourage students to draw a free-body diagram of the plane and use it to write some equations. Some groups will need more assistance than others. Most groups will measure the length of the pendulum (from pivot to center of object). Some groups will measure the radius of the circular motion (from center of circle to center of object); other groups will measure how far below the support point (ceiling) the circle is. Groups need to use this measurement to calculate the vertex angle of the conical pendulum (the angle the string makes with the vertical; see Figure 1). Encourage students to only run the plane when they are making measurements so the battery doesn't run out too quickly — this will help maintain a constant speed for the plane during the experiment.

Once the students have completed their measurements and calculations, they share them with the rest of the class, perhaps using whiteboards or large sheets of paper, for a discussion related to methods of analysis.

## Extension

An extension option is to provide students with AA cells that have different potential differences to power the planes, to first determine whether the potential difference affects speed. Then students can investigate how the speed affects the angle and the radius of the motion for a constant length of string supporting the plane.

## Common Student Challenges

One of the biggest problems students face with circular motion is the idea of centripetal force. Many students seem to think that a “magic” centripetal force is exerted on an object when it is in circular motion, and that the direction of this force is directed outward, not inward to the center of the circle. Students think this because they are confusing centripetal force with inertia. They think that if they were in a car making a fast turn and the door opened, they would be thrown out of the car. Thus, they believe there is a force related to circular motion directed to the outside of the circle.

It is important to emphasize that a force is an interaction between two objects and help students identify the object exerting the force toward the center of another object's circular motion. Ask them to envision that to keep them in the car going around a circle, the door must exert an inward force, since their inertia would cause them to continue moving in a straight line. Trying to make an object, such as a basketball, roll in a circle by only tapping it with a meterstick will also emphasize in which direction the external force must act.



Some teachers go so far as to tell students that there is no such thing as a centripetal force, just like there is no such thing as a down force. The word *centripetal* refers to a direction. Emphasize that some external force, such as the normal force, gravity, friction, or tension must act centripetally to allow an object to execute circular motion. An activity that will help students with this concept is as follows: students draw several free-body diagrams of objects in circular motion and then select (e.g., draw a circle around) the force or forces that act centripetally. In this particular lab, the centripetal direction is horizontally toward the center of the circle in which the plane is flying, so the horizontal component of the tension force is acting centripetally.

Students also need to be reminded, when making measurements of pendulum length ( $L$ ) and radius of the circle ( $R$ ) that the measurements should be made to the center of the moving object. Students often mistakenly take the length of a pendulum as the length of the string or chain supporting the object; however, the pendulum length is from pivot or connection to the center of mass of the pendulum/mass system. If the supporting chain or string has negligible mass, then the pendulum length is measured from the pivot to the center of mass of the object attached.

## Analyzing Results

Ask students to use a stopwatch and compare their calculated period (calculated using the length measurements) to a period measured directly with the stopwatch. They should compute a percent difference between the measured and calculated periods and describe how reasonable their results are. The “measured” period is what students measured with the stopwatch. This should be treated as the theoretical value in this case. The “calculated” period is the one derived from the distance measurements they made. This should be treated as the “experimental” value in their discussion of percent difference. Technically they are both measured values, but in calculating the percent difference, the period measured with the stopwatch has much less uncertainty, and thus can be used to approximate a true value for the period.

Encourage students to consider the uncertainties in their measurements. For example, if they measured the radius of the plane’s motion while it was moving, how precisely could they measure the radius? What is the uncertainty in each of their measurements? What do they think the total uncertainty is? Are their measured and calculated values the same within the limits of precision of their measurements? In other words, is the percent difference between the measured and calculated values less than the total uncertainty in their length measurements?

Students should also consider whether the speed of the plane was actually constant. Ask them how they might have noticed in their data that it wasn’t, and how that would affect their prediction.



Students then use the measurements of varying lengths to determine or verify the relationship between the length of the pendulum and the angle the string makes with the vertical, in order to determine the relationship between the length of the pendulum and the angle the string makes with the vertical as

the object executes circular motion. The equation derived above  $\left(T^2 = \frac{4\pi^2 R}{g \tan \theta}\right)$

relates the radius to the period and the angle. The radius and length of string are related by  $R = L \sin \theta$ . Substitute this into the above equation and one

obtains  $T^2 = \frac{4\pi^2}{g} L \cos \theta$ .

Once students have contemplated these questions within their own groups, then the whole class has a discussion comparing the various groups' methods and which methods were more precise than others. They also present their graphs for comparison and discussion.

## Assessing Student Understanding

After completing this investigation, students should be able to:

- ▶ Draw a free-body diagram of an object moving as a conical pendulum;
- ▶ Design a plan to make measurements to analyze the motion of a conical pendulum;
- ▶ Evaluate the uncertainties in the measurements of length made for a conical pendulum;
- ▶ Use Newton's second law to analyze the motion of a conical pendulum;
- ▶ Predict the period of a conical pendulum using only length measurements;
- ▶ Calculate speed, period, and angle for various lengths; and
- ▶ Graph the relationships and compare them to Newton's second law.

## Assessing the Science Practices

**Science Practice 1.1** The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

<b>Proficient</b>	Draws an accurate picture of the motion of a conical pendulum, and draws an accurate free-body diagram of the conical pendulum.
<b>Nearly Proficient</b>	Draws an accurate free-body diagram of the conical pendulum but adds a fictitious centripetal force to the diagram.
<b>On the Path to Proficiency</b>	Draws an almost accurate free-body diagram with one or more additional forces that are incorrect.
<b>An Attempt</b>	Draws an inaccurate free-body diagram of the conical pendulum.

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**Science Practice 1.4** The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

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<b>Proficient</b>	Makes no mistakes in using a free-body diagram to analyze the motion of a conical pendulum using Newton's second law.
<b>Nearly Proficient</b>	Makes minor mistakes in using the free-body diagram or picture to write equations to analyze the motion of a conical pendulum.
<b>On the Path to Proficiency</b>	Makes major mistakes in using Newton's laws and the free-body diagram to analyze the conical pendulum.
<b>An Attempt</b>	Unable to write equations using the free-body diagram.

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**Science Practice 2.2** The student can *apply mathematical routines* to quantities that describe natural phenomena.

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<b>Proficient</b>	Uses Newton's second law to analyze the motion of a conical pendulum, using length measurements only to calculate period, speed, and angle.
<b>Nearly Proficient</b>	Makes mostly correct calculations from equations; may confuse the use of sine and cosine.
<b>On the Path to Proficiency</b>	Makes some correct calculations from equations.
<b>An Attempt</b>	Makes no correct calculations from equations.

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**Science Practice 4.2** The student can *design a plan* for collecting data to answer a particular scientific question.

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<b>Proficient</b>	Designs an accurate and appropriate plan to make length measurements to predict the period of a pendulum.
<b>Nearly Proficient</b>	Follows directions and adds a design plan that is mostly complete, including diagrams and assumptions.
<b>On the Path to Proficiency</b>	Follows directions but does not clearly indicate a plan for experimental design and procedure.
<b>An Attempt</b>	Misinterprets directions or does not indicate a plan for experimental design and procedure.

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**Science Practice 4.3** The student can *collect data* to answer a particular scientific question.

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<b>Proficient</b>	Collects accurate and appropriate length data from multiple trials using different lengths to predict the period of a conical pendulum.
<b>Nearly Proficient</b>	Collects data that is missing a few minor pieces or is disorganized in its presentation. For example, doesn't perform trials for multiple pendulum lengths.
<b>On the Path to Proficiency</b>	There are major gaps in the data collected, and the presentation lacks any organization.
<b>An Attempt</b>	Collects inaccurate or incomplete data or doesn't provide any organization for this data.

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**Science Practice 5.1** The student can *analyze data* to identify patterns or relationships.

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<b>Proficient</b>	Analyzes the uncertainties in the length measurements made and determines the uncertainty in the period calculated. Graphs the relationships from period, speed, and angle as functions of pendulum length and compares them to Newton's second law.
<b>Nearly Proficient</b>	Estimates uncertainties and calculates a total uncertainty without being clear on how to use this to evaluate the accuracy of the result.
<b>On the Path to Proficiency</b>	Estimates uncertainties in measurements but does not compute a total uncertainty or compare it to the percent difference.
<b>An Attempt</b>	Cannot accurately evaluate the uncertainties in measurements.

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**Science Practice 6.4** The student can *make claims and predictions about natural phenomena* based on scientific theories and models.

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<b>Proficient</b>	Accurately predicts how changing the length of a conical pendulum will change the period of the pendulum and applies this prediction to changes in speed and angle.
<b>Nearly Proficient</b>	Makes a minor mistake in the predictions about the motion of a conical pendulum.
<b>On the Path to Proficiency</b>	Makes only limited predictions about the motion of a conical pendulum.
<b>An Attempt</b>	Cannot make accurate predictions about the motion of a conical pendulum.

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## Supplemental Resources

Butcher, Frank “Circular Motion Studies with a Toy Airplane.” *The Physics Teacher* 25, no. 9 (1987): 572–573.

“Circular Motion.” PBS LearningMedia. Accessed September 1, 2014. <http://www.teachersdomain.org/resource/lsp07.sci.phys.maf.circmotion>. [*This website links to the circular motion videos available on the Rutgers PAER video website and also has background discussions, questions, and standard relations to circular motion. These can be used for a prelab demonstration to show that if an object is moving in a circle, there must be a force directed towards the center of the circle.*]

“The Forbidden F-Word.” The Physics Classroom. Accessed September 1, 2014. <http://www.physicsclassroom.com/class/circles/u6l1d.cfm>. [*This website provides some of the basics of circular motion that can easily be related to curves on roads and highways. The website provides some basic animations as well, and can be used for extensions where students go for further knowledge.*]

“Ladybug Motion 2D.” PhET. University of Colorado Boulder. Accessed September 1, 2014. <http://phet.colorado.edu/en/simulation/ladybug-motion-2d>. [*This simulation provides an interactive way to learn about position, velocity, and acceleration vectors.*]

The Physics Toolbox. Accessed September 1, 2014. <https://www.physicstoolboxinc.com/p-180-centripetal-force-airplane.aspx>.