AP Physics 1 Investigation 2: Newton's Second Law

What factors affect the acceleration of a system?

Central Challenge

In this lab students investigate how the acceleration of an object is related to its mass and the force exerted on the object, and use their experimental results to derive the mathematical form of Newton's second law.

Students should have already completed the study of kinematics and Newton's first law.

Background

Newton's laws are the basis of classical mechanics and enable us to make quantitative predictions of the dynamics of large-scale (macroscopic) objects. These laws, clearly stated in Isaac Newton's *Principia* over 300 years ago, explain how forces arising from the interaction of two objects affect the motion of objects.

Newton's first law states that an object at rest remains at rest, and an object moves in a straight line at constant speed unless the object has a net external force exerted on it.

Newton's second law states that when a next external force is exerted on an object of mass *m,* the acceleration that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force.

The mass of an object in Newton's second law is determined by finding the ratio of a known net force exerted on an object to the acceleration of the object. The mass is a measure of the inertia of an object. Because of this relationship, the mass in Newton's second law is called inertial mass, which indicates how the mass is measured.

Newton's laws of motion are only true in frames of reference that are not accelerating, known as inertial frames.

Real-World Application

There are numerous real-world applications of Newton's second law that can spark student interest. Students can research their favorite sport and apply the concepts learned in this investigation to understand how the magnitude of the acceleration varies when a force is exerted on objects of different mass, such as golf balls, tennis balls, and baseballs.

Another application could be the physics involved when a car encounters ice. Students think the engine makes the car move, but why doesn't it work on ice? It doesn't work because an external force must be exerted on an object by another object to cause acceleration; the tires push back on the ground, the ground pushes forward on the tires, and the car goes forward. Ice interferes with this interaction of external forces on the tires and the ground, and so the wheels just spin.

In this investigation, students use a modified Atwood's machine. Atwood's machines are systems with two masses connected by a cable and pulley, providing for a constant acceleration of any value required (see Figure 1). Some students might be interested in a real-life application of this technology, such as an elevator and its counterweight.

Inquiry Overview

This investigation is structured as a guided inquiry. Students are presented with the question, "What factors affect the acceleration of a system?"

After observing the demonstrations suggested in Part I of the investigation, the students will be guided to discover the factors to be investigated. The students will also design the procedure of the investigation and the data collection strategy.

Students might need some guidance with the analysis of data and the construction of graphs. More specifically, they might be confused about how to merge the results of the two parts of the investigation to answer the overall lab question.

In the Investigation section, specific guiding questions are offered to support students in the design and interpretation of their experiments. Part II of the investigation is divided into two separate activities. The first is limited to the relation of acceleration to force, and the second is limited to the relation of acceleration to mass.

Connections to the AP Physics 1 Curriculum Framework

Big Idea 1 Objects and systems have properties such as mass and charge. Systems may have internal structure.

Enduring Understanding	Learning Objectives
1.A The internal structure of a system determines many properties of the system.	**1.C.1.1** The student is able to design an experiment for collecting data to determine the relationship between the net force exerted on an object, its inertial mass, and its acceleration. (Science Practice 4.2)

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding	Learning Objectives
3.A The internal structure of a system determines many properties of the system.	**3.A.2.1** The student is able to represent forces in diagrams or mathematically using appropriately labeled vectors with magnitude, direction, and units during the analysis of a situation. (Science Practice 1.1)

[**NOTE:** In addition to those listed in the learning objectives above, the following science practices are also addressed in the various lab activities: 4.1, 4.3, 5.1, and 5.3.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.	Students produce multiple representations of the data in the form of graphs and diagrams as follows: • Graphs of the data: › acceleration vs. force › acceleration vs. mass • Force diagrams that represent the forces exerted on the objects
4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.	Students identify the quantities that need to be measured in order to determine the acceleration of the system.
4.2 The student can *design a plan* for collecting data to answer a particular scientific question.	Students design a procedure to investigate the relationships among the net force exerted on an object, its inertial mass, and its acceleration.
4.3 The student can *collect data* to answer a particular scientific question.	Students gather the following data: • net force and acceleration when the total mass is kept constant • total mass and acceleration when the net force is kept constant
5.1 The student *can analyze data* to identify patterns or relationships.	Students analyze the graphs to identify the relationship between the variables
5.3 The student *can evaluate the evidence provided by data sets* in relation to a particular scientific question.	Students articulate an operational definition of Newton's second law based on the evidence presented by the graphs.

[**NOTE:** Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Return to Table of Contents

Equipment and Materials

Per lab group (three to four students):

- Dynamics track
- Cart
- Assorted masses
- Mass hanger and slotted masses
- Low-friction pulley
- String
- Meterstick
- Stopwatch

If you do not have a dynamics track, then any flat, smooth surface, perhaps even the lab tables themselves, will work just fine. The carts should have wheels with a small rotational-inertia and low-friction bearings.

Data acquisition using motion detectors or photogates is recommended when available, as it helps reduce experimental procedural errors. Another option is to record a video of the motion of the cart and use video analysis software to analyze the motion.

Timing and Length of Investigation

- **Teacher Preparation/Set-up:** 15–20 minutes

  This time is needed to prepare the demos and set out equipment from which students may choose for their investigation.

- **Prelab:** 30 minutes

  It is advisable to conduct the activities and prelab discussion in one class or lab period.

- **Student Investigation:** 110–120 minutes

  Design of procedure: 20–30 minutes

  Data collection: 30 minutes

  Data analysis: 60 minutes

  You may assign the design of the data collection procedures as homework. Students gather the materials and do their own setup for their investigations. At the beginning of the lab period, have volunteers present their draft procedures to the class, and solicit feedback from the various groups.

- **Postlab Discussion:** 30 minutes
- **Total Time:** approximately 3.5 hours

  [**NOTE:** This investigation is designed to enable a deeper understanding of Newton's second law and therefore it might take more time than investigations performed in the context of the previous AP Physics B course.]

Safety

There are no major safety concerns for this lab. However, pay attention to high speeds of carts, masses flying off carts, masses hitting the feet of students, and student fingers being squeezed when stopping a cart at the pulley when a high proportion of mass is on the hanger. Also, to keep students and equipment from being damaged, restrict the total slotted mass. General lab safety guidelines should always be observed.

Preparation and Prelab

Prelab Activities

The following activities are optional and could be conducted to assess students' prior knowledge, skill levels, and understanding of key concepts. Setup the modified Atwood machine and pose questions such as those suggested below in this four-part prelab session:

Part I:

What will a graph of the cart's velocity (v) vs. time (t) look like after the system is released from rest?

After making and discussing their predictions, students carry out an experiment, using a motion detector to record v vs. t, or using video capture, in which case students will have to put some thought into how to produce the velocity vs. time graph. But the main point of this part is for students to see and make sense of the conclusion that the slope of the velocity vs. time graph is constant.

Part II:

(a) If the cart's mass is increased, will the new velocity vs. time graph look the same or different from the graph in Part I?

(b) If the hanging mass is increased, will the new velocity vs. time graph look the same or different from the graph in Part I?

Again, these are qualitative questions, but students can obtain quantitative data to answer them. As usual with these kinds of qualitative questions, the lab works well if students first make and discuss their predictions before designing and carrying out the experiments.

Part III:

If both the cart's mass and the hanging mass are doubled, will the new velocity vs. time graph look the same or different from the graph in Part I?

Part IV:
What if the cart is moving initially?

What will the velocity vs. time graph look like, compared to the graph from Part I, if the cart at $t = 0$ is given a brief push away from the pulley? Will the graph be the same? If not, what will be different?

Some students may spontaneously have the idea of doing another trial where the cart is given a brief push towards the pulley — and it would be great for them to try that! They should be able to identify that the y-intercept in the velocity–time graph represents the initial velocity of the cart.

The Investigation

Part I:
In the first part of this activity, students observe a number of demonstrations that include variations of an object being accelerated.

A modified Atwood's machine with a system consisting of a cart and a hanger with slotted masses like the one shown in Figure 1 is a suitable setup.

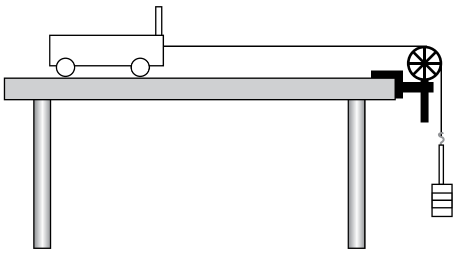

Figure 1

Examples include a demonstration where the total mass of the system is kept constant and the net force is varied, and a demonstration where the net force is kept constant and the total mass of the system is varied. Instructors could use any available lab equipment that allows for a variation of the force exerted on the object with added masses. Ask the students these three questions:

1. "What do you observe?"
2. "What can you measure?"
3. "What can you change?"

A guided discussion should yield some of the following answers to the questions:

1. The cart-mass hanger system is accelerated.
2. Quantities that can be measured include the mass of the cart, the mass of the hanger, distance traveled by the cart, distance traveled by the hanger and the slotted masses, the time of travel, etc.
3. Quantities that can be changed are the net force on the system and the total mass of the system.

Students may have difficulty identifying the net force exerted on the system. Drawing free-body diagrams might help in determining that the net force on the system is equal to the gravitational force of Earth on the hanger and slotted masses. Some students will indicate that a force of kinetic friction is exerted on the cart.

Part II:
After the discussion, instruct students to design two data collection strategies to determine how two factors affect the acceleration of the system: the net force on the system and the total mass of the system.

Activity 1: Students design procedures that include calculation of the acceleration when the total mass of the system is kept constant and the net force is varied.

Activity 2: Students design procedures that include calculation of the acceleration when the total mass of the system is varied and the net force is kept constant.

A few tips:

- Discourage students from trying to combine the two activities into one.
- Encourage students to be careful to keep the string parallel to the track throughout the data collection.
- The length of the string connecting the cart to the mass hanger should allow the mass hanger to reach the floor just before the cart reaches the pulley.
- Make sure that the string does not rub against anything, such as the pulley mount.

Extension

An extension to this lab is to investigate the effect of friction on the acceleration of the cart. Alternative investigations that use dynamics concepts can be provided as challenges. For examples of this type of activity, see "Turning a Common Lab Exercise into a Challenging Lab Experiment: Revisiting the Cart on an Inclined Track" and "Time Trials — An AP Physics Challenge Lab" in Supplemental Resources.

Another engaging extension activity consists of having students apply the concepts learned in this investigation to their favorite sports. Students could do short presentations in the class, or they could create a poster with their findings if time for presenting is a constraint.

The Science360 Video Library, sponsored by the National Science Foundation, gathers the latest science videos by scientists, colleges and universities, and science and engineering centers. "Newton's Three Laws of Motion" and "Science of the Summer Olympics: Engineering In Sports" are recommended for students to explore (see Supplemental Resources).

Return to
Table of Contents

Common Student Challenges

Some of the common challenges that students have regarding Newton's first law include the idea that forces are required for motion with constant velocity. When observing the demonstrations, students need to recognize that the velocity of the object is changing as a result of the net force exerted on the object. It should be clear that the net force determines an object's acceleration, not its velocity. To counter this student misconception, you can use a motion detector and a force probe to study the motion of a cart being pulled by a mass hanging from a string that passes over a pulley (as shown in the Investigation section). Simultaneously graph the force on the cart and the motion of the cart. Direct students to notice the shape of the force graph (horizontal line) and acceleration graph are the same, but the velocity vs. time graph is a line with a positive slope. A constant forward force produces an increasing velocity and a constant acceleration.

Students might not see the connection between Newton's laws and kinematics, so it is important for them to recognize Newton's second law as "cause and effect." It is important to present Newton's second law in its operational form of $\vec{a} = \frac{\Sigma \vec{F}}{m}$, as the commonly used $\sum \vec{F} = m\vec{a}$ leads some students to believe that the product of mass and acceleration (ma) is a force.

A specific student challenge in this investigation is to recognize that both the cart and the falling mass are included in the total inertial mass of the system being affected by the gravitational force on the falling mass. During the investigation, all masses to be used as falling masses should be placed in the cart when not pulling the cart. Students will be tempted to have the cart on the table and replace the falling mass with a different falling mass that is on the lab table. This, in effect, changes the total mass being pulled. This is a good opportunity to have students discuss the meaning of *system*. The system that is being accelerated is the cart and falling mass.

Another specific student challenge is the role of friction of the cart and the pulley as well as the rotational inertia of the wheels of the cart and the pulley. These can be ignored when conducting the investigation for sufficient hanging mass, but should be discussed at some point in the analysis of results.

Analyzing Results

How students analyze their results depends on how they decided to make measurements and complete the calculations. Some students may use a stopwatch to measure the time of the acceleration over a fixed distance. These students would then use the equations of constant acceleration motion to calculate the acceleration. Other students may choose to use motion sensors to plot the velocity vs. time for the cart. In that case, they would use the slope of the graph for the acceleration.

The sources of experimental uncertainty depend on the equipment used as the precision is limited by the apparatus resolution. In this investigation, uncertainty might be related to the measurements of time, length, or mass (or combinations of each). Students can minimize the uncertainties by taking measurements in multiple trials and averaging the results. See Resources for options of support in this area.

The development of mathematical models from graphs of acceleration vs. force and acceleration vs. mass are an expectation of this investigation. In order to determine the relationship between net force and acceleration and between total mass and acceleration, students plot a graph with an independent variable on the horizontal axis and a dependent variable on the vertical axis. If students are not familiar with linearization methods, guide them as they linearize the acceleration vs. mass graph.

The use of multiple representations in this lab is highly recommended as it leads to a deeper conceptual understanding of Newton's second law. The lab report should include verbal descriptions of their observations as well as labeled free-body diagrams of the forces exerted on the system.

Sample qualitative graphs for this lab include:

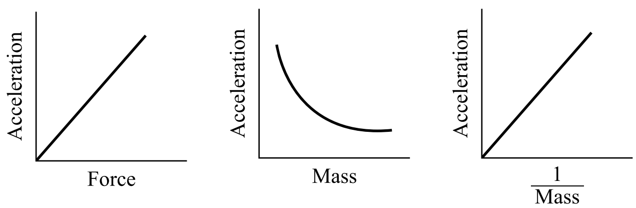

Figure 2

Following are several guiding questions that will help students interpret their graphs generated in Part II of the investigation:

Activity 1:

How does your data indicate if the acceleration was proportional to the force?

Students determine the relationship between the acceleration and the force from the graph. A straight line represents a direct variation between the acceleration and the net force.

What does the slope of the acceleration vs. force graph represent?

The slope of the acceleration vs. force graph represents the reciprocal of the mass of the system.

What is the algebraic relationship between acceleration and net force in this system?

The algebraic relationship between acceleration and net force is expressed as $a \propto \Sigma F$.

[**NOTE:** You may want to point out to students that the graph does not go through zero. This accounts for the frictional force between the cart and the surface.]

AP PHYSICS 1 INVESTIGATIONS

Activity 2:

How does the data indicate if the acceleration was inversely proportional to the mass?

Students determine the relationship between the acceleration and the mass from the graph. A hyperbola represents an inverse variation between the acceleration and the mass.

What does the slope of the acceleration vs. the inverse of the mass represent?

The slope of the acceleration vs. the inverse of the mass graph represents the net force of the system.

What is the algebraic relationship between acceleration and mass in this system?

The algebraic relationship between acceleration and mass is expressed as $a \propto \frac{1}{m}$.

As part of the analysis, students could find the percent difference between the theoretical value of the acceleration from one configuration of the masses using the free-body diagram of the system and the experimental value.

[**NOTE:** Percent difference is applied when comparing two experimental quantities, E1 and E2, neither of which can be considered the "correct" value. The percent difference is the absolute value of the difference over the mean times 100.]

Assessing Student Understanding

By the end of the investigation, students should be able to:

- Articulate that the acceleration of an object is directly proportional to the net force: $a \propto \Sigma F$;
- Articulate that the acceleration is inversely proportional to the mass: $a \propto \frac{1}{m}$;
- Determine a relationship between arbitrary combinations of mass, force, and acceleration using dimensional analysis;
- Calculate the proportionality constant (k) for the relationship derived from dimensional analysis: $a = k\frac{\Sigma F}{m}$;
- Obtain a proportionality constant value of 1.0; and
- Identify the sources of experimental uncertainty and ways to minimize experimental uncertainties.

Return to
Table of Contents

Assessing the Science Practices

Science Practice 1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

Proficient	Creates accurate and appropriate graphical representations of the relationship between acceleration and net force and between acceleration and mass.
Nearly Proficient	Creates mostly correct graphical representations of the relationship between acceleration and net force and between acceleration and mass. The graphs may not fully reflect all aspects of the relationships among the variables.
On the Path to Proficiency	Creates flawed or incomplete graphical representations of the relationship between acceleration and net force and/or between acceleration and mass.
An Attempt	Provides incorrect graphical representations of the relationship between acceleration and net force and/or between acceleration and mass.

Science Practice 4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.

Proficient	Provides accurate and detailed justification explaining the relevance of the variation of mass and net force in the system.
Nearly Proficient	Provides accurate justification for the relevance of the variation of mass and net force in the system with only an occasional or minor error.
On the Path to Proficiency	Provides justification for the relevance of the variation of mass and/or net force in the system with occasional and/or minor errors; justification may be correct but lacks completeness.
An Attempt	Provides generally weak justification for the relevance of the variation of mass and/or net force in the system justification; includes minimal reasoning and evidence.

Return to Table of Contents

Science Practice 4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

Proficient	Designs an effective data collection plan to answer the question via well-selected quantitative measurements of acceleration, providing rationales for all choices. Accurately evaluates uncertainty in measurements. Effectively explains equipment selection for acquiring data (balance and meterstick and stopwatch or motion detector or photogates). Accurately explains different sources of error in data. Accurately identifies and explains independent, dependent, and controlling variables, and justifies choices as follows: (1) Determination of the acceleration when the total mass of the system is kept constant and the net force is varied. (2) Determination of the acceleration when the total mass of the system is varied and the net force is kept constant.
Nearly Proficient	Designs an appropriate data collection plan to answer the question via quantitative measurements of acceleration; measurements may lack complete details. Identifies equipment (balance and meterstick and stopwatch or motion detector or photogates). Identifies appropriate data sources and estimated error. Accurately identifies and describes independent, dependent, and controlling variables as follows: (1) Determination of the acceleration when the total mass of the system is kept constant and the net force is varied. (2) Determination of the acceleration when the total mass of the system is varied and the net force is kept constant.
On the Path to Proficiency	Designs a data collection plan to answer the question via quantitative measurements of acceleration; measurements may not be clearly defined or articulated. Acknowledges need to consider estimated error. Accurately identifies independent, dependent, and controlling variables with few errors as follows: (1) Determination of the acceleration when the total mass of the system is kept constant and the net force is varied. (2) Determination of the acceleration when the total mass of the system is varied and the net force is kept constant.
An Attempt	Presents an incomplete data collection plan to answer the question. Makes errors in identifying the variables (independent, dependent, and controlling).

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects appropriate data to fully determine the relationship among the acceleration, net force, and inertial mass of the system with precision of observations, accuracy of records, and accurate use of scientific tools and conditions. Accurately applies mathematical routines and appropriately uses measurement strategies.
Nearly Proficient	Collects appropriate and adequate data to answer some aspects of the relationship among the acceleration, net force, and inertial mass of the system with only minor errors in the precision of observation, record keeping, and use of tools and conditions. Selects appropriate mathematical routines and provides measurements with only few minor errors.
On the Path to Proficiency	Collects appropriate data to determine the relationship among the acceleration, net force, and inertial mass of the system. Provides observation logs and record keeping that contain several errors. Selects appropriate mathematical routines and provides measurements with few errors or only a single significant error.
An Attempt	Collects relevant but significantly inadequate data to determine the relationship among the acceleration, net force, and inertial mass of the system. Provides observations and/or record keeping that are incomplete and/or inadequate for answering a particular question. Selects inappropriate mathematical routines; measurements contain many errors.

Return to
Table of Contents

Science Practice 5.1 The student can *analyze data* to identify patterns or relationships.

Proficient	Comprehensively describes the patterns and relationships within data relative to the relationship among the acceleration, net force, and inertial mass of the system. Accurately applies appropriate mathematical routines. Correctly identifies all of the sources of experimental error, and suggests ways to minimize the uncertainties.
Nearly Proficient	Identifies most patterns within data relative to the relationship among the acceleration, net force, and inertial mass of the system with only an occasional minor error. Selects appropriate mathematical routines and applies them with only minor errors. Correctly identifies most of the sources of experimental error, and suggests ways to minimize the uncertainties.
On the Path to Proficiency	Identifies the most obvious patterns within data, relative to the relationship among the acceleration, net force, and inertial mass of the system with some errors and inaccuracies. Selects appropriate mathematical routines but makes some application errors. Identifies some of the sources of experimental error, and suggests ways to minimize the uncertainties.
An Attempt	Identifies a few legitimate patterns in data, though these may be irrelevant to determine the relationship among the acceleration, net force, and inertial mass of the system. Identifies some mathematical routines that are appropriate. Identifies some of the sources of experimental error, but does not suggest ways to minimize the uncertainties.

Science Practice 5.3 The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.

Proficient	Provides a connection along with a clear justification, such as the calculation of the proportionality constant (k), for the relationship derived from dimensional analysis to determine the relationship between the acceleration and the inertial mass of the system and the relationship between the acceleration and the net force of the system.
Nearly Proficient	Provides a connection but no justification is offered, or a justification is offered but it is vague regarding the relationship between the acceleration and the inertial mass of the system and/or the relationship between the acceleration and the net force of the system. Attempts to represent the proportionalities among acceleration, net force, and inertial mass as an equation; rearranges and solves for the constant of proportionality k.
On the Path to Proficiency	Provides a connection but the generalization of the relationship between the acceleration and the inertial mass of the system and/or the relationship between the acceleration and the net force of the system is not correct.
An Attempt	Fails to recognize or provide a connection to the relationship between the acceleration and the inertial mass of the system, and the relationship between the acceleration and the net force of the system.

Return to
Table of Contents

Supplemental Resources

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