AP Physics 1 Investigation 1: 1D and 2D Kinematics

How is the translational motion of a ball described by kinematics?

Central Challenge

Students observe a steel ball rolling down an inclined ramp, then across a horizontal track, and finally as a projectile off the end of the ramp onto the floor. In the three parts of this investigation, they are tasked with describing, with graphs and equations, the motion of the ball on the inclined ramp, the horizontal track, and as a projectile.

Background

The complete description of motion includes a discussion of the position, velocity, and acceleration of an object at each point in time. The displacement of an object is the change in its position. The velocity of an object is the rate of change of its position. Velocity includes not only the magnitude of that rate of change but also the direction. The acceleration is the direction and rate of change of the velocity of the object.

These relationships can be represented graphically. The velocity can be obtained by finding the slope of the graph of position as a function of time. The acceleration can be obtained by finding the slope of the graph of velocity as a function of time. The critical concepts are contained in the equations for motion with constant acceleration in one dimension, as follows:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$$
Equation 1

$$v_x = v_{x0} + a_x t$$
Equation 2

In these equations, x is the position at time t and x_0 is the position at time t = 0 of the object; v_x is the velocity of the object along the direction of motion, x, at time t, and v_{x0} is the velocity of the object along the direction of motion, x, at time t = 0; and a_x is the acceleration of the object along the direction of motion, x.

Real-World Application

Kinematics is present in many aspects of students' lives, such as driving or riding in automobiles and the sports they play. Driving involves acceleration in linear motion. Even the timing of traffic lights depends on kinematics; in order to keep traffic flowing efficiently, civil engineers need to time red lights at sequential cross streets so that cars aren't stopped at each light, and on roads with higher speed limits they must extend the duration time of yellow lights so that drivers are able to stop safely before the light turns red. Examples of kinematics in sports include cross-country running, which involves constantspeed motion, distance, and displacement; and the motion of a volleyball, which can be approximated using projectile motion.

Inquiry Overview

This multipart inquiry-based investigation introduces students to concepts in kinematics in one and two dimensions. Students perform three guided-inquiry investigations that involve the study of constant velocity (Part I), constant acceleration (Part II), and projectile motion (Part III), which simultaneously involves constant velocity horizontally and constant acceleration vertically.

Through guided inquiry, students are provided with a track that includes an inclined section and a horizontal section. The students are tasked to determine if the motion on the horizontal section is constant velocity and if the motion on the inclined section is constant acceleration. They are then asked to determine how the initial velocity of the ball in projectile motion affects its horizontal motion from the time it leaves the track until it lands on the ground.

Connections to the AP Physics 1 Curriculum Framework

Big Idea 3 The interactions of an object with other objects can be described by forces.

Enduring Understanding	Learning Objectives
3A All forces share certain common characteristics when considered by observers in inertial reference frames.	3.A.1.1 The student is able to express the motion of an object using narrative, mathematical, and graphical representations. (Science Practices 1.5, 2.1, and 2.2)
	3.A.1.2 The student is able to design an experimental investigation of the motion of an object. (Science Practice 4.2)
	3.A.1.3 The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. (Science Practice 5.1)

[NOTE: In addition to those listed in the learning objectives above, Science Practice 4.3 is also addressed in this investigation.]

Skills and Practices Taught/Emphasized in This Investigation

Science Practices	Activities
1.5 The student can re-express key elements of natural phenomena across multiple representations in the domain.	Students use data from the different parts of the investigation to create graphs of the motions and write equations that relate to those motions as part of the analysis of their lab.
2.1 The student can justify the selection of a mathematical routine to solve problems.	Students select appropriate equations to describe the ball's motion in either constant velocity, constant acceleration, or projectile motion as part of the analysis of the lab.
2.2 The student can apply mathematical routines to quantities that describe natural phenomena.	Students use data they have collected in the appropriate equations; they also construct graphs from data to describe various motions.
4.2 The student can <i>design a plan</i> for collecting data to answer a particular scientific question.	Student groups, using the equipment provided, design a plan to collect enough data to plot the motions and to make calculations related to the motions, enabling them to determine which parts of the motion are constant velocity, constant acceleration, or projectile motion.
4.3 The student can <i>collect</i> data to answer a particular scientific question	Students collect displacement and time measurements to plot graphs of position vs. time or velocity vs. time.
5.1 The student can <i>analyze data</i> to identify patterns or relationships.	Students analyze the data they gather to make calculations and graphs to determine which parts of the motion are constant velocity, constant acceleration, or projectile motion. For example, they use the slope of the position–time graph to determine velocity and compare that to the velocity–time graph and calculations for the same part of the motion.

[NOTE: Students should be keeping artifacts (lab notebook, portfolio, etc.) that may be used as evidence when trying to get lab credit at some institutions.]

Equipment and Materials

Per lab group (two students):

- Ramp attached to a horizontal track (see below for one possible way to construct a ramp; if you choose a different type of track, make certain that the steel ball follows a straight-line path and does not veer off the track, as this will make data collection impossible)
- Stopwatch
- Meterstick
- ▶ Steel ball (1.5–2 cm in diameter)
- Carbon paper

- Bubble level
- (Optional) Toy car that accelerates

The ramps are constructed from aluminum sliding door C-channel, and they can be built for approximately \$10 per lab station from materials that are readily available at local home-improvement stores.

Per ramp:

- ▶ One 2-foot piece of 1/2-inch aluminum C-channel
- ▶ One 2-foot piece of 3/8-inch aluminum C-channel
- Two 6-inch pieces of aluminum C-channel (preferably 1 inch wide, but scraps will do)
- ► Two #6-32 × 1/2-inch machine screws
- Two nuts to fit the machine screws

To construct four ramps:

Get two 8-foot lengths of C-channel, one 1/2-inch wide to form the horizontal tracks at the base of the ramps and one 3/8-inch wide to form the inclined sections of the ramps. The bottom end of the 3/8-inch piece used for the upper, angled part of each ramp fits snugly into the upper end of the 1/2-inch horizontal track piece. Also purchase one piece of wider C-channel to cut into short sections to attach for "feet."

Cut the ½-inch C-channel into four 2-foot lengths with a hacksaw or band saw to make the four horizontal sections. Cut the smaller 3/8-inch C-channel into four 2-foot lengths to make the four upper track pieces that will be angled.

Two feet are needed for each ramp. The feet can be made from larger or leftover C-channel turned upside down under the track piece so the nuts on the bottom fit inside the channel and attach to the ramp pieces with machine screws and nuts. Drill two 3/16-inch holes in each section of the C-channel, 6-8 inches from the ends. Attach the feet to the wider C-channel with the machine screws (wing nuts are preferable, but any #6-32 nut will do). It is very important that the screws be set so that they in no way interfere with the path of the ball. To make each foot, turn the short piece of 1-inch (or scrap) C-channel upside down under the track and attach the two together with the screws and nuts.

Duct tape or a C-clamp can be used to fasten the ramp and track to the table so that repeated trials are consistent and not affected by changing the elevation of the upper track. With this design, the inclined piece of C-channel is movable (necessary to perform the exercise in Part III of this investigation) since one end can be elevated to different heights with small wooden blocks.

Another option is to construct the tracks to be twice as long (i.e., with a 4-foot lower section and 4-foot upper section); these are harder to store, but they provide more length on which students can take measurements. Just double the cut lengths in the directions above to accomplish this.

Figure 1 is a good picture of what the C-channel looks like, how the feet are attached, and how it should be supported.



Figure 1

Figure 2 shows how the narrower piece of channel fits into the wider piece of channel to provide a smooth transition from the angled ramp part of the track to the horizontal section.

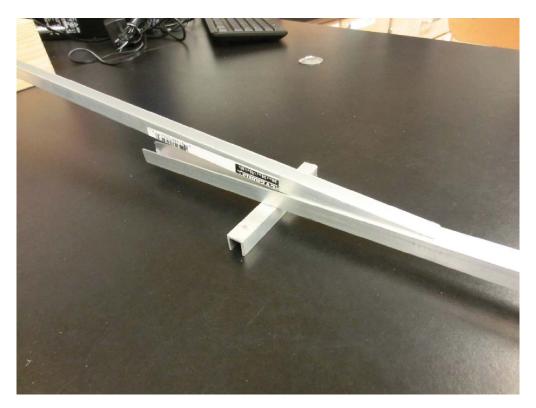


Figure 2

Alternate equipment ideas:

- ▶ Use 6-foot lengths of flexible vinyl threshold, which is also available from local home-improvement stores. These provide an ideal track for tennis balls and are very inexpensive. The inclined ramp portion would need to be supported by a board, as it is flexible and will move if unsupported as the tennis ball rolls along it. The tennis balls will not make a mark on the carbon paper so other methods would need to be used to determine the landing point of the projectile. [NOTE: It is important that ramps are grooved so that the ball moves in a straight motion down the ramp without veering or falling off.]
- Commercially made ramps are also available from popular scientific equipment companies. These are, however, significantly more expensive, and in some of them the flat, horizontal section and the inclined section are all one piece, so the angle of incline is fixed. These do not offer students the flexibility of changing the incline.
- If the technology is available, give students photogates and the computer interfaces necessary to operate them. Avoid giving students motion detectors, however. They should be required to take simple displacement and time measurements to make their conclusions in this activity.

Timing and Length of Investigation

► Teacher Preparation/Set-up: 10–15 minutes

The ramps are light and can be setup in at most 10 minutes. This time does not include construction of the ramp itself, which should take 20–30 minutes per ramp.

Student Investigation: 70–80 minutes

Allow students time to observe the ramp, play with releasing the ball and watching it move along the track, and for small-group discussion in groups of a few lab pairs so that they can determine what they will measure and how they will measure those quantities as they approach each of the three parts to this investigation. Obtaining the data should take 10 minutes or less for each exercise and 20–30 minutes to conduct the multiple trials required for Part III.

Postlab Discussion: 15–20 minutes

▶ **Total Time:** approximately 1.5–2 hours

Safety

There are no specific safety concerns for this lab; however, all general lab safety guidelines should be followed. Sometimes, if the aluminum has been cut, the elevated end can be a little sharp — put a cushion on the elevated end, such as a foam ball, to protect students' faces.

Preparation and Prelab

This activity should come after students work with motion detectors (or other motion analysis methods) to learn about graphs of motion and after you have helped them derive the equations of constant acceleration motion from the graphs of motion. Students should also be familiar with graphing techniques and creating graphs of position vs. time and velocity vs. time prior to the lab. Some activities are available in "Special Focus: Graphical Analysis" (see Supplemental Resources).

It is also useful to have students understand a little bit about measuring time with a stopwatch and the size of reaction-time uncertainties. You may want to have them time one oscillation of a short pendulum and compare measurements to compute an uncertainty. Then have several students in the class time one oscillation of a long pendulum (2 meters or more) and compare measurements. They should see that the percent uncertainty of the timing of the long pendulum is much less than the percent uncertainty for the short pendulum. This is true even though the absolute time uncertainty may be about the same. Reinforce for them the idea that, in order to reduce uncertainty, they need to time the motion over longer distances whenever possible.

This experiment uses a rolling ball, so the motion description is only for linear (or translational) motion. Since a portion of the ball-Earth system's original gravitational potential energy is converted to rotational kinetic energy of the ball, the ball's linear speed on the horizontal portion of the track will be less than predicted by conservation of energy; also, the distance from the track that the ball lands on the floor will be less than predicted. Students will not yet have studied rotational kinematics, but it will not be difficult for them to understand that part of the system's initial energy goes to rotational kinetic energy so that the ball has less linear (or translational) speed on the level track and as a consequence less range when it flies off onto the floor. If students have discussed rotational motion prior to this lab, they should record this and discuss it in their laboratory report as both an assumption and a source of uncertainty. Otherwise, you might not need to even address the conservation of energy or rotational motion; the data could be revisited when rotational motion is covered, to calculate the predicted distance including the rotational energy, and compare with the experimental observations.

The Investigation

The following set of lab exercises provides an introduction to kinematics in one and two dimensions without the use of expensive sensors or low-friction tracks and carts. The exercises are all built around the ramp.

The three parts to this investigation involve:

- 1. The study of one-dimensional accelerated motion of the ball in its direction of motion down the incline;
- 2. A study of constant velocity one-dimensional motion along the horizontal portion of the track; and
- 3. A study of two-dimensional motion as the ball leaves the table.

Part I: Constant Velocity

The goal of the first part of this lab is for students to devise a plan to determine whether the motion on the horizontal portion of the track is constant-velocity motion. They can be given as much or as little instruction as you see fit. Instruct students to only to use stopwatches and metersticks and to present their results to the class at the end of the investigation and defend their answers.

Hopefully students will remember that a graph of constant velocity motion is a straight line with non-zero slope on a position vs. time graph, or a horizontal line on velocity vs. time graph and choose to create a graph of position vs. time or velocity vs. time. However, expect students' creativity to prevail and several methods to emerge — both valid and invalid. The onus remains on students to justify why their chosen method is valid.

Conducting a class discussion at the end of this portion of the lab before proceeding to the next is optional. If you notice that several groups are headed in the wrong direction, you may wish to redirect their efforts in a class discussion before proceeding to Part II.

Part II: Constant Acceleration

The goal of the second exercise is for students to design an experiment to determine if the motion of the ball down the ramp is one of constant acceleration. This is more challenging for students. Since you are not directly telling students what to measure, they may need several chances to fail before they find the right measurements that will yield a valid claim about the motion of the ball.

Challenge students to present an analysis of the motion that justifies their claim that it is constant acceleration. Some students will recall that the graph of position vs. time for a constant acceleration motion is a parabola. However, it will be difficult for students to prove that the graph is a parabola unless they are familiar with curve-fitting programs on their calculator or a computer. In this case, you may choose to guide students to the realization that a plot of displacement vs. the square of time should yield a straight line with a slope of $\frac{1}{2}a_x$ for the motion on the inclined ramp, and therefore justifies their claim about the motion.

Students may choose to plot a graph of velocity vs. time. Experience has shown that students tend to think they can calculate the velocity at any point by dividing the distance traveled by the time. Remind students that this is the average velocity over that interval and not the instantaneous velocity at the end of the interval.

Also remind them that they are not to assume that the acceleration is constant. You might need to stop the entire class to have them debrief and share measurement techniques if they head off in the wrong direction. They are to use data to demonstrate that acceleration is constant without necessarily finding its value. Students should not be allowed to use the equations of constant acceleration to prove the acceleration is constant. They must use a position vs. time graph or velocity vs. time graph.

Part III: Projectile Motion

The goal of the last part of the investigation is to provide students with an introduction to projectile motion. Ask the students to determine how the initial velocity of a projectile launched horizontally affects the distance it travels before it strikes the ground. Their experiments in Part I will prepare them to measure several different velocities for the ball as it leaves the track. The ball rolls off the end of the track and strikes the ground a distance from where it left the track. Give students as much direction as you want on how to reliably measure the x component of the displacement (the horizontal distance it travels). They likely have not had experience with carbon paper, so you may need to explain to them how it works: a steel ball landing on the paper will cause a dot to appear on a piece of paper placed under the carbon side of the paper.

Once students have displacement data for several different values of launch velocity, they use a graph to determine the relationship between the two variables. Once you have discussed the equations of constant acceleration applied to projectile motion, students refer back to their graph and how it supports the mathematical derivations.

Extension

One possible extension for this lab is to challenge students to plot the vertical motion of the ball in projectile motion as a function of time. You can give them as much or as little direction as you want. Students know the horizontal speed of the projectile as it leaves the track. If they place a vertical board in the path of the ball with the carbon paper attached, the ball will strike it and the vertical height at that location can be measured. They then move the board away from the launch point in fixed intervals and record the vertical position of the ball for a series of horizontal distances.

The analysis of this is somewhat more complicated because students tend to confuse the horizontal and vertical motions and analyze the two together. A class discussion should lead them to the conclusion that, since the velocity in the horizontal direction is constant, the various equally spaced vertical-board positions represent equal time measurements; and thus a position vs. time graph can be obtained.

Another possible extension is to provide students with a toy car that accelerates and have them determine if the acceleration is constant, and if so, how long the acceleration lasts. (Arbor Scientific and other companies sell cars they market as "constant acceleration" cars.) Instruct students to support or refute the validity of their claim with data, graphs, and calculations.

Common Student Challenges

It is essential for this lab that students are comfortable graphing position and velocity as functions of time.

If they still have difficulties with this, then you may want to take them outside and have them time the motion of students walking and running. Have students with stopwatches stand at 5-meter intervals along a straight line, and direct them to start timing when a student starts moving, and stop timing when the student passes them. The data of position vs. time is shared with the whole class. Students could then graph the data as practice for this lab.

A common student mistake is to assume they can apply the equations of constant acceleration to determine if an object executes constant acceleration motion. Experience has shown that students will study various sections of a larger motion and use the equations of constant acceleration to calculate the acceleration. They will then compare the various accelerations to determine if the acceleration is constant over the whole range of motion. For example, they will use the equations of constant acceleration to calculate the acceleration for the first 10 centimeters, then the first 20 centimeters, then the first 30 centimeters, etc.; then they will compare these to determine if the acceleration was constant. How long to allow students to pursue this incorrect path is up to you. You may decide to circulate amongst the groups and ask each what their plan is, and have individual discussions about the validity of their plans. Or you may choose to hold a class discussion after all of the groups have made some progress. In either case, if they choose this incorrect method, direct students to create and use graphs of position vs. time or instantaneous velocity vs. time.

Students should use boxes or books to elevate the end of the ramp to change the acceleration and therefore the final horizontal velocity of the ball. They can use a piece of carbon paper taped to a piece of white paper on the floor to precisely determine the point of impact of the ball. Not allowing too great an incline keeps the velocity low so that the ball only travels about 30–35 centimeters in the horizontal direction after falling from the average 80-centimeter lab table.

Another challenge is the concept of rotational motion of the ball (discussed above), which students will not completely understand at this point. It is enough here for them to know that the rolling motion of the ball accounts for a different kind of kinetic energy (rotational) but the velocity they are calculating from linear kinetic energy is only part of the total energy. However, if energy has not yet been discussed in class, then students may not even worry about the rolling motion. [NOTE: Discourage students from attempting to use conservation of energy calculations during this investigation to determine the final horizontal velocity of the ball: it does not address the learning objectives in this investigation.]

Analyzing Results

Whether students break for a discussion of the results after each section of the lab or only at the end is up to you. It is highly recommended, however, that the discussion of the measurement of the velocity as it leaves the track is discussed prior to starting Part III.

The most convincing arguments for constant velocity involve a graph of position vs. time. Students should be able to articulate how they made the measurements that construct the graph. Some students may have measured the speed at different locations on the track and compared the values to each other. The discussion should center on the validity of the measurements: whether, in fact, they measured displacement and time. Depending on how the large the displacement is, the velocity they calculated may be an average velocity and not an instantaneous velocity. This discussion provides an excellent opportunity to reinforce the difference between the two.

The most convincing arguments for constant acceleration involve a graph of velocity vs. time or a graph of displacement vs. time squared. Both of these will yield a straight-line graph if the acceleration is constant. As mentioned above, the common misconception here is for students to confuse average velocity and instantaneous velocity. Experience has shown that students will measure the time it takes for the ball to roll significant distances (30–50 centimeters), measure the time, and then divide one by the other. They assume this is the velocity at the end of the motion rather than the average velocity. It is important to help students realize that this is not the case and how to calculate the instantaneous speed (which is the same size as the instantaneous velocity, since the ball does not change direction of motion).

The analysis of Part III is also best done using a graph. Ask the students to consider the following questions:

- How did you measure the speed of the ball just before it left the track?
- How consistent was the landing position of the ball for each individual speed?

- What does the shape of the graph of horizontal displacement vs. speed imply about the relationship between the two?
- How does the ball's time of flight depend on its initial horizontal speed?
- ▶ How could you improve the precision and accuracy of your measurements?

A discussion of sources and sizes of uncertainty of measurements is inevitable in this lab. Start by having students indicate what measurements were actually made and what the uncertainty was in each measurement. For example, they will probably measure time with a stopwatch. If they measure several trials, then they can take a standard deviation; otherwise the uncertainty is their reaction time.

Depending on the incline of the track, the speed of the ball may be significant, making timing with a stopwatch significantly affected by reaction-time error. Methods of decreasing this uncertainty can be discussed at any point during the measurement or in a discussion at the end. Ask the students to consider the following questions:

- What is the typical human reaction time when using a stopwatch?
- ▶ How does this time compare to the time intervals you were measuring?
- What percent uncertainty does this introduce into your time measurements and speed calculations?
- What could you do to reduce this uncertainty?

For example, a typical reaction time is between 0.1 and 0.25 seconds. Assuming the larger value, if the measurement is only 1.0 second, this represents a 25 percent uncertainty in the timing measurement. However, if the time measurement is 10 seconds, this represents a 2.5 percent uncertainty in the timing measurement and thus the speed measurement. One suggestion for reducing uncertainty would be to use a device that does not rely on human reaction time for measurement, such as a photogate.

Assessing Student Understanding

After completing this investigation, students should be able to:

- Use measurements of displacement and time to create a position vs. time graph;
- Use measurements of displacement and time to create a velocity vs. time graph;
- Use graphs of position and velocity vs. time to analyze the motion of an object;
- Determine the speed of a ball on a horizontal track;
- Measure the horizontal distance a projectile travels before striking the ground; and
- ▶ Relate the initial velocity of a horizontally launched projectile to the horizontal distance it travels before striking the ground.

Assessing the Science Practices

Science Practice 1.5 The student can re-express key elements of natural phenomena across multiple representations in the domain.

Proficient	Plots correct graphs for all parts of the motion, and makes
	correct inferences about the motion from those graphs.

Nearly Proficient Plots correct graphs for all parts of the motion, but

portions of the interpretation are incorrect.

On the Path to Proficiency

Plots a correct graph for one part of the motion (e.g.,

the velocity vs. time for the level section).

An Attempt Attempts graphs related to his or her observations

and measurements, but graphs are inaccurate.

Science Practice 2.1 The student can *justify the selection of a mathematical routine* to solve problems.

Proficient Uses kinematic equations appropriately to verify

displacement, velocity, and acceleration for all sections of the experiment, including correct interpretations of slope.

Nearly Proficient In most instances, uses correct equations for calculations related

to motion, but there is an incorrect assumption in one step, such as forgetting that initial vertical velocity as the ball leaves the table is zero. This applies also to determination of slope and area from graphs.

On the Path to Proficiency Uses some correct equations for calculations, but uses one or more incorrectly, such as using a kinematics equation to determine whether acceleration is constant. This applies also to determination of slope and area from graphs..

An Attempt

Nearly Proficient

Uses incorrect equations to calculate acceleration, velocity, and/or displacement, and uses incorrect equations in determination of slope and area from graphs.

Science Practice 2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

ProficientMakes entirely correct calculations from equations or determinations of slope and area from graphs.

Makes mostly correct calculations from equations or

determinations of slope and area from graphs.

On the Path to Makes some correct calculations from equations or Proficiency determinations of slope and area from graphs.

An Attempt Attempts to make calculations from equations or determinations of slope and area from graphs, but none are correct.

Science Practice 4.2 The student *can design a plan* for collecting data to answer a particular scientific question.

Proficient	Follows directions and adds a thorough description
	of a design plan (with clearly labeled diagrams),
	including predictions and assumptions.
Nearly Proficient	Follows directions and adds a design plan that is mostly
	complete (with diagrams), and including assumptions.
On the Path to	Follows directions but does not clearly indicate a
Proficiency	plan for experimental design and procedure.
An Attempt	Misinterprets directions or does not indicate a viable
	plan for experimental design and procedure.

Science Practice 4.3 The student can *collect data* to answer a particular scientific question.

Proficient	Collects accurate data in a methodical way and presents the data in an organized fashion.
Nearly Proficient	Collects mostly but not entirely accurate and complete data or the presentation of the data is somewhat disorganized.
On the Path to Proficiency	Collects somewhat inaccurate or incomplete data and the presentation of the data lacks organization.
An Attempt	Collects inaccurate or incomplete data and doesn't provide any organization for this data.

Science Practice 5.1 The student can analyze data to identify patterns or relationships.

Proficient	Appropriately uses a velocity–time graph to determine the acceleration of the ball and position–time graphs to determine the speed of the ball on the track. Accurately graphs horizontal displacement vs. speed and interprets the results.
Nearly Proficient	Makes conclusions and calculations from data (perhaps graphs) but indicates no clear correlations.
On the Path to Proficiency	Requires significant assistance in analyzing velocity–time graphs or relating horizontal distance traveled for a projectile launched horizontally to the initial speed of the projectile.
An Attempt	Attempts to use incorrect features of a velocity–time graph to determine the acceleration of an object.

Supplemental Resources

Drake, Stillman. *Galileo: Two New Sciences*. Madison, Wisconsin: University of Wisconsin Press, 1974.

"Mechanics: 1-Dimensional Kinematics." The Physics Classroom. Accessed September 1, 2014. http://www.physicsclassroom.com/calcpad/1dkin/problems. cfm. [This website allows students to explore extra practice problems on kinematics.]

"The Moving Man." PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/en/simulation/moving-man. [This simulation provides an interactive way to learn about position, velocity, and acceleration graphs.]

The Physlet Resource. Davidson College. Accessed September 1, 2014. http://webphysics.davidson.edu/physlet_resources. [This resource provides sample "physlet" illustrations, explorations, and problems in 1-dimensional kinematics.]

"Projectile Motion." PhET. University of Colorado Boulder. Accessed September 1, 2014. http://phet.colorado.edu/en/simulation/projectile-motion. [Provides multiple visual representations of kinematics in one and two dimensions.]

"Special Focus: Graphical Analysis." AP Physics 2006–2007 Professional Development Workshop Materials. College Board. Accessed September 1, 2014. http://apcentral.collegeboard.com/apc/public/repository/AP_Physics_Graphical_Analysis.pdf.